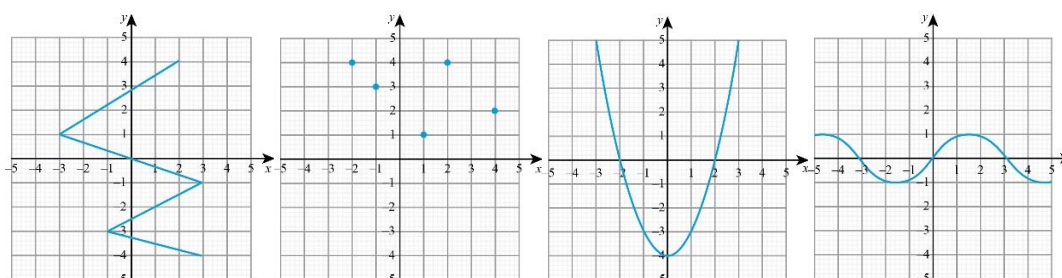


# 2 Functions

## Activity: Functions test (Teacher version)

1 In the space below list **ALL** relations (from A to G) that are **functions**.

B, C, D, F, G



A

B

C

D

E  $\{(2, 3), (2, 4), (7, 10)\}$

F  $\{(3, 0), (4, 0), (5, 0), (6, 1)\}$

G

$x$	$y$
1	1
2	8
3	27
4	64

2 Given  $f(x) = 4x + 2$  and  $g(x) = x^2 - 4x + 3$ , evaluate:

a)  $f(5) =$

$$\begin{aligned} f(5) &= 4(5) + 2 \\ &= 22 \end{aligned}$$

b)  $g(-1) =$

$$\begin{aligned} g(-1) &= (-1)^2 - 4(-1) + 3 \\ &= 8 \end{aligned}$$

3 Given  $f(x) = 5x + 1$  and  $g(x) = x^2 - 3x + 1$ , find  $g(f(x))$  then expand and simplify.

$$g(f(x)) = (5x + 1)^2 - 3(5x + 1) + 1$$

$$g(f(x)) = 25x^2 + 5x - 1$$

4 Given  $f(x) = 3x^2 + 2$ , find:

a)  $f^{-1}(x)$

$$y^2 = \frac{x-2}{3}$$

$$y = \pm \sqrt{\frac{x-2}{3}}$$

$$f^{-1}(x) = \pm \sqrt{\frac{x-2}{3}}$$

b)  $f^{-1}(f(x))$

$$\text{If } f^{-1}(x) = +\sqrt{\frac{x-2}{3}} \text{ then } f^{-1}(f(x)) = x$$

$$\text{If } f^{-1}(x) = -\sqrt{\frac{x-2}{3}} \text{ then } f^{-1}(f(x)) = -x$$

5 a) Express the radius  $r$  of a circle as a function of the circumference  $C$ .

$$r = \frac{C}{2\pi}$$

b) Express the area  $A$  of the circle as a function of  $C$ .

$$A = \pi r^2$$

$$A = \frac{C^2}{4\pi}$$

6 Let  $f(x) = \sqrt{x}$ ,  $g(x) = 6x - 3$ , and  $h(x) = \frac{x}{3}$ . Find  $h(f(g(x)))$ .

$$h(f(g(x))) = \frac{\sqrt{6x-3}}{3}$$

7 Find the inverse of the following function.

$$f(x) = \sqrt{\frac{x-2}{x+1}}$$

$$x = \sqrt{\frac{y-2}{y+1}}$$

$$f^{-1}(x) = \frac{-x^2 - 2}{x^2 - 1}$$

8 A balloon is inflated in such a way that its volume increases at a rate of  $20 \text{ cm}^3 \text{ s}^{-1}$ .

- a) Given that the volume of the balloon was  $100 \text{ cm}^3$  when the process of inflation began, what will the volume be after  $t$  seconds of inflation? *Use function notation.*

$$V(t) = 20t + 100$$

- b) Assuming that the balloon is a sphere while it is being inflated, express the radius  $r$  of the balloon as a function of  $t$ .

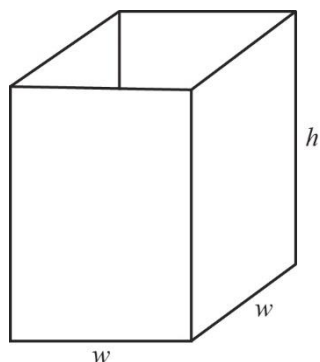
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
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$$20t + 100 = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{15t+75}{\pi}}$$

$$r(t) = \sqrt[3]{\frac{15t+75}{\pi}}$$

- 9 An open-topped box with a square base is to be constructed from sheet metal in such a way that the completed box is made of  $2 \text{ m}^2$  of sheet metal. Express the volume of the box as a function of the base width.



$$V = w^2 h$$

$$\text{Area of sheet metal} = 2 \text{ m}^2$$

$$\text{Leading to } w^2 + 4wh = 2$$

Substitution leads to:

$$V = w^2 \left( \frac{2 - w^2}{4w} \right)$$

$$V(w) = \frac{2w - w^3}{4}$$